PROVISIONAL ANSWER KEY
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Note:
1). All Suggestions are to be sent with reference to website published Question paper with Provisional Answer Key Only.
2). All Suggestions are to be sent in the given format only.
3). Candidate must ensure the above compliance.

101. The number of groups of order 4 are:

(A) 1 (B) 2

(C) 3 (D) 4

102. If we denote the n^{th} term of the sequence 1/2, 2/3, 3/4,... by $\{a_n\}$, which one of the following is incorrect

(A) $a_n = \left\{\frac{n}{n+1}\right\}, n \geq 1$

(B) $a_n = \left\{\frac{n+1}{n+2}\right\}, n \geq 0$

(C) $a_n = \left\{\frac{n-9}{n-8}\right\}, n \geq 10$

(D) $a_n = \left\{\frac{n-5}{n-4}\right\}, n \geq 5$

103. For each of the following sequences, which one of them is strictly increasing

(A) $\{\cos(1/n)\},\ n \geq 1$

(B) $\{n^2 - n\},\ n \geq 0$

(C) $\{n(n-2)\},\ n \geq 0$

(D) $\{\log_e(1/n)\},\ n \geq 1$

104. A field with four number of elements is given by the following:

(A) $\mathbb{Z}_4$

(B) $\mathbb{Z}_4 \times \mathbb{Z}_2$

(C) $\mathbb{Z}_2 \times \mathbb{Z}_4$

(D) None of these

105. Which of these sequences is bounded above? In each case $n \geq 0$ if it makes sense, otherwise $n \geq 1$

(A) $\{(-1)^n/n\}$

(B) $\{\sqrt{n}\}$

(C) $\{e^n\}$

(D) $\{\log_e n\}$

106. What is the limit of the increasing sequence $a_n = (n-1)/n,\ n \geq 1$?

(A) 1/2

(B) 2/3

(C) 1

(D) ∞

107. What is the limit of the increasing sequence $a_n = \cos(1/n),\ n \geq 1$?

(A) 1/2

(B) 2/3

(C) 1

(D) ∞

108. Which of the following options are correct for the statements I and II?

I. Every cyclic group is Abelian.

II. Every Abelian group is cyclic.

(A) I only.

(B) I and II only.

(C) II only.

(D) None of the above.

109. What is the limit of the increasing sequence $a_n = 1 + 1/2 + 1/4 + ... + 1/2^n$?

(A) 3

(B) 2

(C) 1

(D) ∞

110. In order to prove that the sequence $a_n = \left(1 + \frac{1}{2^n}\right)^{2^n}$ has a limit, it suffices to prove that a_n is

(A) either increasing or bounded above.

(B) increasing and bounded above.

(C) either decreasing or bounded below.

(D) decreasing and bounded below.

111. Which one of the following statement is true about a monotone sequence $\{a_n\}$?

(A) it is increasing for all n.

(B) it is decreasing for all n.

(C) if it is bounded it has a limit.

(D) all of the above.

112. Which one of the following is not a bounded monotone sequence

(A) $1/n$

(B) $\sin(1/n)$

(C) $\log_e(1/n)$

(D) $(-1)n$

113. If $|a| \geq 2$ and $|b| \leq 1/2$, then $|a + b|$ has a lower estimate as

(A) 1/2

(B) 1

(C) 3/2

(D) 2

114. The limit of $a_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx$ (as $n \to \infty$) is

(A) 0

(B) $\pi/2$

(C) π

(D) ∞

115. The limit of $a_n = \int_0^{\frac{\pi}{2}} x^n (1-x)^n \, dx$ (as $n \to \infty$) is

(A) 0

(B) $\log_e n$

(C) e^n

(D) ∞

116. Which one of the following is true about $\Sigma 1/n^p$

(A) converges if $p > 1$

(B) diverges if $p \leq 1$

(C) both A and B are true

(D) A is true and B is false

117. Which one of the following diverges

(A) $\sum_{n=2}^{\infty} \frac{1}{n^3 - 2n + 1}$

(B) $\sum \sqrt{\frac{4n}{n^2 + 1}}$

(C) $\sum \sqrt{\frac{n}{n^2 - 4}}$

(D) $\sum \frac{n}{n^2 + 1}$

118. Which one of the following converges

(A) $\sum \frac{(-1)^n}{\sqrt{(n)}}$ (B) $\sum (-1)^n \frac{n}{n+2}$

(C) $\sum (-1)^n \frac{\cos n\pi}{n}$ (D) None of the above

119. Which one of the following is true about $\sum_{n=1}^{\infty} \frac{x^{2n}}{2^n n}$

(A) converges for $|x| < \sqrt{2}$ (B) diverges for $|x| > \sqrt{2}$

(C) both A and B are true. (D) None of the above.

120. Which one of the following is true about the function $f(x) = |x|, x \in \mathbb{R}$?

(A) is increasing

(B) is decreasing

(C) neither increasing nor decreasing

(D) data is insufficient to conclude

121. Which one of the following is true about the function $f(x) = x^n, n \in \mathbb{N}$ and $x \in \mathbb{R}$?

(A) is increasing.

(B) is decreasing.

(C) neither increasing nor decreasing.

(D) data is insufficient to conclude.

122. Laplace's equation in two variables $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is classified as

(A) Elliptic. (B) Hyperbolic.

(C) Parabolic. (D) None of these.

123. The kernel K(x, y) of the integral equation $f(x) = \phi(x)\lambda \int_0^x e^{x-y} \phi(y)\, dy$ is given by

(A) e^{x+y} (B) e^{x-y}

(C) e^{-x-y} (D) None of these.

124. The solution of the integral equation $y(x) = x + \int_0^x (x-t)\, y(t)\, dt$ is

(A) $\frac{1}{2}(e^x - e^{-x})$

(B) $\frac{1}{2}(e^x + e^{-x})$

(C) $\frac{1}{2}(\cos x - \sin x)$

(D) None of these.

125. The maximum of xy^2z^2 subject to the constraint $x + y + z = 12$ is

(A) (0,0,0)

(B) $\left(\frac{12}{5}, \frac{24}{5}, \frac{24}{5}\right)$

(C) $\left(\frac{12}{5}, \frac{24}{5}, \frac{12}{5}\right)$

(D) data is insufficient to conclude.

126. The functional $I(y) = \int_0^1 y(x)\, dx$ has minimal value for the following function y:

(A) $y = x^3$

(B) $y = x^2$

(C) $y = x$

(D) $y = e^x$

127. Which one of the following are semi–infinite closed intervals

(A) $(a, b], [a, b)$

(B) $[a, \infty), (-\infty, a]$

(C) $(a, \infty), (-\infty, a)$

(D) $(-\infty, \infty)$

128. Which one of the following is false about the improper integral $\int_0^\infty e^{kx}\, dx$ where $x \in \mathbb{R}$?

(A) converges to -1 for $k < 0$

(B) converges to $-1/k$ for $k < 0$

(C) diverges for $k > 0$

(D) diverges for $k = 0$

129. Which one of the following improper integrals diverges?

(A) $\int_{2+}^{4} \frac{dx}{\sqrt{x-2}}$

(B) $\int_0^{1-} \frac{dx}{1-x^2}$

(C) $\int_0^\infty \frac{x^2\, dx}{1+x^3}$

(D) $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\, dx$

130. Which one of the following is false about the sequence of functions $f_n(x) = \frac{1}{n}\sin(nx)$ where $f(x) = \lim_{n\to\infty} f_n(x)$?

(A) $f(x)$ is a continuous function

(B) $\lim f_n(x)$ converges uniformly to the function $f(x)$

(C) $f'(x) = \lim \; f_n'(x)$

(D) the sequence of functions $f_n''(x)$ does not converge

131. The residue of the complex valued function $f(z) = -\frac{1}{z}$ at the origin is

(A) 0 (B) 1

(C) -1 (D) $\frac{1}{2}$

132. The radius of the convergence of the power series $f(z) = \sum_{n=0}^{\infty} z^{n^2}$, where $z \in \mathbb{C}$ is

(A) 0 (B) 1

(C) 4 (D) $\frac{1}{2}$

133. Suppose (X, d) and (Y, e) are metric spaces and $\phi : X \to Y$. Then ϕ is called an isometry or isometric map if, and only if,

(A) $e(\phi(a), \phi(b)) = d(a, b) \forall a, b \in X$

(B) $e(\phi(a), b) = d(\phi(a), b) \; \forall a, b \in X$

(C) $e(a, b) = d(\phi(a), \phi(b)) \; \forall a, b \in X$

(D) None of the above

134. If X is a metric space and A and B are subsets of X for which $A \subseteq B$ then

(A) diam(A) < diam (B) (B) diam(A) ≤ diam (B)

(C) diam(A) > diam (B) (D) diam(A) ≥ diam (B)

135. Suppose S is a subset of $\mathbb{R}$ Then

(A) diam(S) = sup(S) – inf(S) (B) diam(S) = sup(S) + inf(S)

(C) diam(S) = sup(S) × inf(S) (D) diam(S) = sup(S) / inf(S)

136. Suppose X is a metric space. The distance from any point of X to a non–empty subset of X is non negative real number. Since, we define $\inf(\phi)$ to be ∞, then it follows that

(A) $\text{dist}(x, \phi) = 0 \ \forall x$.
(B) $\text{dist}(x, \phi) = \infty \ \forall x$.
(C) $\text{dist}(x, \phi)$ does not exist.
(D) data insufficient to conclude.

137. Suppose $x \in \mathbb{R}$ and $x \in \mathbb{R}^+$ and $\mathbb{Q} \cap (x, x + r) \neq \phi$, it follows that

(A) $\text{dist}(x, \mathbb{Q}) < r$
(B) $\text{dist}(x, \mathbb{Q}) = 0$
(C) $\text{dist}(x, \mathbb{R}\backslash\mathbb{Q}) < r$
(D) All of the above.

138. Suppose S is a subset of $\mathbb{R}$, then which one of the following is a false statement?

(A) $\text{dist}(z, S) \leq |z - \sup S|$ with equality if $z \geq \sup S$
(B) $\text{dist}(z, S) \geq |z - \inf S|$ with equality if $z \leq \inf S$
(C) if $\sup S \in \mathbb{R}$, then $\text{dist}(\sup S, S) = 0$
(D) if $\inf S \in \mathbb{R}$, then $\text{dist}(\inf S, S) = 0$

139. Suppose X is a metric space, $x \in X$ and A and B are non empty subsets of X for which $A \subseteq B$, then which of the following statement is true?

(A) $\text{dist}(x, B) \leq \text{dist}(x, A)$
(B) $\text{dist}(x, A) \leq \text{dist}(x, B) + \text{diam}(B)$
(C) $\text{diam}(B) \geq \text{diam}(A)$
(D) All of the above

140. Which of the following statement is false for the set $\mathbb{Z}_n$?

(A) $\mathbb{Z}_n$ is a cyclic group
(B) $\mathbb{Z}_n$ is a field if and only if n is prime number
(C) $\mathbb{Z}_n$ is an Abelian group
(D) None of these

141. Suppose X is a metric space, $z \in X$ and S is a subset of X. Then z is called an accumulation point (also known as limit point) of S in X, if and only if,

(A) $\text{dist}(z, X) = 0$
(B) $\text{dist}(z, X\backslash\{z\}) = 0$
(C) $\text{dist}(z, S) = 0$
(D) $\text{dist}(z, S\backslash\{z\}) = 0$.

142. Which of the following is a false statement?

(A) every point in $\mathbb{R}$ is an accumulation point (also known as limit point) of $\mathbb{Q}$

(B) every point in $\mathbb{R}$ is an accumulation point of $\mathbb{R}$

(C) every point in $\mathbb{Q}$ is an accumulation point of $\mathbb{R}$

(D) None of the above

143. Suppose X is a metric space, $z \in X$, S is a subset of X, acc(S) and iso(S) represent the set of accumulation points and isolated points of S respectively. Which of the following is incorrect statement?

(A) If $z \notin X$, then $z \in$ acc(S) if, and only if, dist(z, S) $\neq 0$

(B) If $z \in X$, then $z \in$ acc(S) if, and only if, $z \notin$ iso(S)

(C) $z \in$ acc(S) if, and only if, $z \notin$ iso(S) and dist(z, S) $= 0$

(D) None of the above

144. Suppose X, d is a metric space, $x \in$ X and A and B are subsets of X. Then

(A) dist(A, B) > dist(x, A) + dist(x, B)

(B) dist(A, B) > dist(x, A) – dist(x, B))

(C) dist(A, B) $\leq$ dist(x, A) + dist(x, B)

(D) None of the above

145. Suppose X is a metric space, S being a subset of X, S^c being complement of S in X and $a \in$ X. Then a is called a boundary point of S in X if and only if,

(A) dist (a, S) = 0

(B) dist(a, S^c) =0

(C) both A and B are true

(D) either A or B is true

146. Suppose X is a metric space, S being a subset of X, Sc being complement of S in X. The collection of boundary points of S in X is called the boundary of S in X denoted by dS. Which of the following is incorrect statement?

(A) dist (a, S) = 0 $\forall a \in \partial S^c$

(B) dist (a, S^c) = 0 $\forall a \in \partial S$

(C) $\partial S^c = \partial S$

(D) None of the above

147. Suppose X is a metric space, S being a subset of X, S^c being complement of S in X. The collection of boundary points of S in X is called the boundary of S in X denoted by ∂S. The collection of accumulation point of a set A is represented by acc(A). Which of the following is incorrect statement?

(A) If $a \notin S$ then $a \in \partial S$ if, and only if, $a \notin$ acc(S)

(B) If $a \in S$ then $a \in \partial S$ if, and only if, $a \in$ acc(S^c)

(C) If $a \notin S$ then $a \in \partial S$ if, and only if, dist(a, $S \backslash \{a\}$) = 0

(D) None of the above

148. Suppose X is a metric space, $S \subseteq X$, S^c being complement of S in X. $\overline{S}$ and S° being closure and interior of S respectively. Which of the following is incorrect statement?

(A) $\overline{S} = \{x \in X \mid \text{dist}(x, S) \neq 0\}$

(B) the exterior of S is $\{x \in X \mid \text{dist}\ (x, S) > 0\}$

(C) $S^\circ = \{x \in X \mid \text{dist}\ (x, S^c) > 0\}$

(D) None of the above

149. Suppose (X, d) is a metric space, $w \in$ X and $A \subseteq X$. The collection of boundary points of A in X denoted by ∂A. $\overline{A}$ being closure of A. Which of the following is incorrect statement?

(A) diam $(\overline{A})$ = diam(A)

(B) dist $(w, A) \leq$ dist$(w, \partial A)$

(C) dist $(w, \overline{A})$ = dist(w, A)

(D) None of the above

150. Suppose X is a metric space. The collection of boundary points of S in X is denoted by ∂S. $\overline{S}$ and S° being closure and interior of S respectively. Which of the following is incorrect statement?

(A) $\partial \overline{S} \subseteq \overline{S}$

(B) $\partial(S^\circ) \cup S^\circ \neq \phi$

(C) $\overline{\overline{S}} = \overline{S}$

(D) $(S^\circ)^\circ = S^\circ$

151. The two matrices A = $[a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{p \times q}$ are equal when

(A) $m = p$

(B) $n = q$

(C) $a_{ij} = b_{ij} \forall i, j$

(D) All of the above

152. Which of the following statement is false about the matrices?

(A) The addition of two matrices is commutative

(B) The subtraction of two matrices is associative

(C) The addition of two matrices is associative

(D) None of the above

153. Given any $m \times n$ matrices A and B, the matrix equation $3(X + 0.5A) = 5(X - 0.75B)$ has a solution

(A) $X = 0.75A + 1.175B$

(B) $X = 0.75A + 1.875B$

(C) $X = 0.175A + 1.125B$

(D) $X = 0.25A + 0.75B$

154. Which of the following is false about matrix multiplication?

(A) The matrix multiplication of any three matrices is associative

(B) The matrix multiplication of any two matrices is commutative

(C) The matrix multiplication of two symmetric square matrices is commutative.

(D) None of the above

155. If $\lambda \in \mathbb{R}$ and A and B be real matrices. Which of the following operation is not always true about a transpose of a matrix.

(A) $(A')' = A$

(B) $(A + B)' = A' + B'$

(C) $(\lambda A)' = \lambda A'$

(D) $(AB)' = A'B'$

156. Which of the following statement is true about a square matrix A?

(A) $A + A'$ is symmetric

(B) $A - A'$ is skew–symmetric

(C) A can be expressed as sum of a symmetric and skew–symmetric matrix

(D) All of the above

157. Suppose A and B be the matrices of size $n \times n$ with A being symmetric and B being skew–symmetric. Which of the following is skew–symmetric?

(A) A^2

(B) B^2

(C) $AB + BA$

(D) $AB - BA$

158. If A and B are 2×2 matrices then the sum of the diagonal elements of $AB - BA$ is

(A) greater than zero
(B) less than zero
(C) equal to zero
(D) data insufficient to conclude.

159. If $A = \begin{bmatrix} \cos\eta & \sin\eta \\ -\sin\eta & \cos\eta \end{bmatrix}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then AB is

(A) $\begin{bmatrix} \cos(\eta+\theta) & \sin(\eta+\theta) \\ -\sin(\eta+\theta) & \cos(\eta+\theta) \end{bmatrix}$
(B) $\begin{bmatrix} -\cos(\eta+\theta) & \sin(\eta+\theta) \\ -\sin(\eta+\theta) & \cos(\eta+\theta) \end{bmatrix}$
(C) $\begin{bmatrix} -\cos(\eta+\theta) & \sin(\eta+\theta) \\ -\sin(\eta+\theta) & -\cos(\eta+\theta) \end{bmatrix}$
(D) $\begin{bmatrix} -\cos(\eta+\theta) & -\sin(\eta+\theta) \\ -\sin(\eta+\theta) & -\cos(\eta+\theta) \end{bmatrix}$

160. If A and B are $n \times n$ matrices and the Lie product is given as $[AB] = AB - BA$ then which of the following is generally not true?

(A) $[[AB]C] = [A[BC]]$
(B) $[[AB]C] + [[BC]A] + [[CA]B] = 0$
(C) $[(A + B)C] = [AC] + [BC]$
(D) None of the above

161. If the product of two numbers is equal to zero then one of them has to be zero. This rule holds in which of the following algebraic structure?

(A) $\mathbb{Z}_2$
(B) $\mathbb{Z}_4$
(C) $\mathbb{Z}_6$
(D) $\mathbb{Z}_8$

162. If $Ax = b$ is a linear system of equation with A being coefficient matrix, x being the unknown vector and b being the given right hand side vector, then which of the following operation will change the solution (sequence of the elements of solution vector is not important)?

(A) interchange of two rows
(B) multiply a row by a non–zero scalar
(C) add one row to another
(D) None of the above

163. The maximum number of linearly independent columns or rows in the matrix $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 2 & 3 & 3 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ is

(A) 1 (B) 2

(C) 3 (D) 4

164. The row rank of the matrix $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 1 & 4 \\ 0 & -1 & -2 \\ 0 & 1 & 2 \end{bmatrix}$ is

(A) 1 (B) 2

(C) 3 (D) 4

165. The row rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 1 & \lambda & 1 & 1 & 4 \\ 1 & 1 & \lambda & 3-\lambda & 6 \\ 2 & 2 & 2 & \lambda & 6 \end{bmatrix}$ when $\lambda = 1$ and $\lambda = 2$ is

(A) 1 and 2 (B) 2 and 3

(C) 2 and 4 (D) 4 and 4

166. If A is an $m \times n$ matrix then the homogeneous system of equations $Ax = 0$ has a non trivial solution if and only if

(A) rank $(A) = n$ (B) rank $(A) > n$

(C) rank $(A) < n$ (D) None of the above

167. If r is the rank of the matrix $\begin{bmatrix} 1 & \alpha & 0 & 0 \\ -\beta & 1 & \beta & 0 \\ 0 & -\gamma & 1 & \gamma \\ 0 & -\delta & 1 & \delta \end{bmatrix}$ then which of the following is a correct statement?

(A) $r > 1$

(B) $r = 2$ if and only if $\alpha\beta = -1$ and $\gamma = \delta = 0$

(C) $r = 3$ if and only if either $\gamma = \delta$ or $\alpha\beta = -1$ and γ, δ are both non–zero.

(D) All of the above

168. Which of the following is a correct statement about the matrix $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 2 & 3 & 5 & 8 \\ 1 & 4 & 5 & 9 \end{bmatrix}$?

(A) The matrix has only left inverse
(B) The matrix has only right inverse
(C) The matrix has both left and right inverse
(D) The matrix has neither left nor right inverse

169. If a matrix M has both left inverse X and right inverse Y then necessarily
I M is square
II $X = Y$
(A) I only
(B) II only
(C) either I or II
(D) both I and II

170. If M is a full rank matrix of size $n \times n$ then which of the following is a correct statement?
(A) M has a left inverse
(B) M has a right inverse
(C) M is of rank n
(D) All of the above

171. If A and B are $n \times n$ matrices and product AB is invertible then which of the following is a correct statement?
(A) A is invertible and nothing can be said about B.
(B) B is invertible and nothing can be said about A.
(C) both A and B are invertible.
(D) Data insufficient to conclude.

172. Which of the following are subspaces of the vector space $\mathbb{R}^{n \times n}$?
(A) the set of symmetric matrices of size $n \times n$.
(B) the set of invertible matrices of size $n \times n$.
(C) the set of non–invertible matrices of size $n \times n$.
(D) all of the above.

173. Which of the following are the basis for $\mathbb{R}^3$?
(A) {(1,1,1), (1, 2, 3), (2,–1,1)}
(B) {(1,1, 2), (1, 2, 5), (5, 3, 4)}
(C) both A and B
(D) None of them

174. In the vector space $\mathbb{R}^4$ let A = span{(1, 2, 0, 1), (–1, 1, 1, 1)} and B = span{(0, 0, 1, 1), (2, 2, 2, 2)} then the dimension of the space $A \cap B$ is
(A) 0
(B) 1
(C) 2
(D) 3

175. If V is of dimension n then which of the following is a correct statement?

(A) every subset of V containing more than n elements is linearly dependent.

(B) No subset of V containing fewer than n elements spans V.

(C) both (A) and (B) options

(D) None of the above.

176. If W is a proper subspace of a finite–dimensional vector space V then

(A) $\dim(W) = \dim(V)$

(B) $\dim(W) > \dim(V)$

(C) $\dim(W) < \dim(V)$

(D) Data insufficient to conclude.

177. If W is a subspace of a finite–dimensional vector space V and $\dim(W) = \dim(V)$ then

(A) $W \subseteq V$

(B) $V \subseteq W$

(C) $V = W$

(D) All of the above.

178. Which of the following is a subspace of $\mathbb{R}^3$?

(A) the zero set $\{(0, 0, 0)\}$

(B) any line passing through origin.

(C) any plane passing through the origin

(D) All of the above

179. The subspace $\{(x, x, x); x \in \mathbb{R}\}$ of $\mathbb{R}^3$ is of dimension

(A) 0

(B) 1

(C) 2

(D) 3

180. Which of the following is not a subspace of $\mathbb{R}^4$?

(A) $\{(a, b, c, d); a + b = c + d\}$

(B) $\{(a, b, c, d); a + b = 1\}$

(C) $\{(a, b, c, d); a^2 + b^2 = 0\}$

(D) None of the above

181. Which of the following mappings $f : \mathbb{R}^3 \to \mathbb{R}^3$ is not linear?

(A) $f(x, y, z) = (y, z, 0)$

(B) $f(x, y, z) = (z, -y, x)$

(C) $f(x, y, z) = (x-1, x, y)$

(D) $f(x, y, z) = (x+y, z, 0)$

182. Suppose $B \in \mathbb{R}^{n\times n}$. Which of the following mappings $T_B : \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$ is not linear?

(A) $T_B(X) = XB - BX$

(B) $T_B(X) = XB^2 + BX$

(C) $T_B(X) = XB^2 - BX^2$

(D) None of the above

183. If $f : V \rightarrow W$ is linear then which of the following statement is true?

(A) f is injective
(B) Kernel of f is $\{0\}$
(C) both A and B options
(D) None of the above

184. The linear mapping $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by
$f(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$

(A) is surjective
(B) is injective.
(C) is bijective
(D) is neither surjective nor injective.

185. The linear mapping $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by
$f(x, y, z) = (x + y + z, 2x - y - z, x + 2y - z)$

(A) is surjective
(B) is injective
(C) is bijective
(D) is neither surjective nor injective

186. If V and W be the vector spaces each of the dimension n over a field. If $f : V \rightarrow W$ is linear then

(A) f is injective
(B) f is surjective
(C) f is bijective
(D) All of the above

187. Two linear mappings $f, g : V \rightarrow W$ are equal if and only if

(A) Kernel of f = Kernel of g
(B) Image of f = Image of g
(C) $f(v_i) = g(v_i)$ for every basis element v_i.
(D) None of the above.

188. The rank and the dimension of the null space of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 2 \\ 0 & 1 & -2 \end{bmatrix} = 0 \text{ is}$$

(A) 3 and 0
(B) 2 and 1
(C) 1 and 2
(D) 0 and 3

189. If A and B be subspaces of finite–dimensional vector space V. The smallest subspace of V that contains $A \cup B$ is given by $A + B = \{a + b;\ a \in A,\ b \in B\}$. Then

(A) $\dim(A + B) = \dim(A) + \dim(B) - \dim(A \cap B)$
(B) $\dim(A + B) = \dim(A) + \dim(B) + \dim(A \cup B)$
(C) $\dim(A + B) = \dim(A) + \dim(B)$
(D) $\dim(A + B) = \dim(A) - \dim(B)$

190. The solution of the equation $\det\begin{bmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{bmatrix} = 0$ is

(A) $x = a$ and $x = 3a$
(B) $x = -a$ and $x = -3a$
(C) $x = -a$ and $x = +3a$
(D) $x = a$ and $x = -3a$

191. If the linear system of equation $Ax = b$ has two distinct solution x_1 and x_2. Which of the following statement is necessarily true?

(A) A is invertible
(B) $x_1 = -x_2$
(C) There exist a solution such that $x \neq x_1$ and $x \neq x_2$
(D) $b = 0$

192. The solution of the system $ax + by - z = 1$, $x - ay - az = -1$ and $ax - y + az = 1$ is $(x, y, z) = (a, b, a)$. If a is not an integer, what is the numerical value of $a + b$?

(A) $-3/2$
(B) -1
(C) 0
(D) 1/2

193. If A, B and $C \in \mathbb{R}^{2\times 2}$, then which of the following statement is true?

I. $A^2 = 0 \Rightarrow A = 0$
II. $AB = AC \Rightarrow B = C$
III. A is invertible and $A = A^{-1} \Rightarrow A = I$ or $A = -I$

(A) I only
(B) I and III only
(C) III only
(D) None of the above

194. Solve for n $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^n - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$

(A) $n = 6$
(B) $n = 5$
(C) $n = -5$
(D) $n = 7$

195. If the inverse of the matrix $\begin{bmatrix} 3 & -2 & -2 \\ -1 & 1 & 1 \\ 3 & -1 & -2 \end{bmatrix}$ is $\begin{bmatrix} 1 & a & 0 \\ -1 & b & 1 \\ 2 & c & -1 \end{bmatrix}$ then

(A) 2
(B) 3
(C) -3
(D) -2

196. The vectors $v_1 = (-1, 1, 1)$, $v_2 = (1, 1, 1)$, and $v_3 = (1, -1, k)$ form a basis for $\mathbb{R}^3$ for all real values of k except

(A) $k = -2$
(B) $k = -1$
(C) $k = 0$
(D) $k = 1$

197. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ then det (A) – rank (A) is

(A) – 2
(B) – 1
(C) 1
(D) 2

198. If det $\begin{bmatrix} a & b & c \\ k & l & m \\ p & q & r \end{bmatrix} = d$ then det $\begin{bmatrix} k & 2(a-k) & p+k \\ l & 2(b-l) & q+l \\ m & 2(c-m) & r+m \end{bmatrix}$ is equal to

(A) – 8d
(B) – 2d
(C) 2d
(D) 8d

199. The value of x for which the matrix $\begin{bmatrix} 7 & 6 & 0 & 1 \\ 5 & 4 & x & 0 \\ 8 & 7 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ cannot be inverted is

(A) –1
(B) 0
(C) 1
(D) 3

200. The value of x for which the vector $\begin{bmatrix} 12 \\ 11 \\ x \end{bmatrix}$ is in the column space of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is

(A) – 26
(B) – 10
(C) 10
(D) 26

201. What is the dimension of the space spanned by the column vectors of the following matrix? $\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$

(A) 2
(B) 3
(C) 4
(D) 5

202. The dimension of the subspace of real symmetric matrices of size $n \times n$ in the space of all real matrices of size $n \times n$ is

(A) $n/2$

(B) n^2

(C) $1/2\,(n(n - 1))$

(D) $1/2\ (n(n + 1))$

203. The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that maps (1, 2) to (–1, 1) and (0, –1) to (2, –1) will map (1, 1) to

(A) (1, 0)

(B) (1, 2)

(C) (2, 1)

(D) (–1, 0)

204. If A is and invertible matrix with eigenvalue λ corresponding to eigenvector x then which of the following statement is true?

(A) The matrix A^{-1} has eigenvalue $1/\lambda$ corresponding to eigenvector x.

(B) The matrix A^{-1} has eigenvalue $1/\lambda$ corresponding to eigenvector whose elements are given by the reciprocal of the elements of x.

(C) The matrix A^2 has eigenvalue 2λ corresponding to eigenvector x.

(D) None of the above.

205. The eigenvalues of $\begin{bmatrix} 2 & b \\ 3 & -1 \end{bmatrix}$ are -4 and $b - 1$ where

(A) $b = 3$

(B) $b = 4$

(C) $b = 5$

(D) $b = 6$

206. The matrix $\begin{bmatrix} 2 & 2+i \\ 2-i & 6 \end{bmatrix}$ has an eigenvalue as

(A) 3

(B) 7

(C) i

(D) $1 + i$

207. If the variables P, V and R are related by equation $PV = nRT$, where n and R are constants then $\frac{\partial V}{\partial T}\ \frac{\partial T}{\partial P}\ \frac{\partial P}{\partial V}$

(A) nR

(B) $1/nR$

(C) $-\ 1$

(D) RT

208. Suppose $f(x, y) = xy(x^2 - y^2)/(x^2 - y^2)$ when $(x, y) \neq (0, 0)$ and $f\ (x, y) = 0)$ when $(x, y) \neq (0, 0)$. The value of f_{xy} at origin is

(A) -1

(B) 0

(C) 1/2

(D) undefined

209. A right circular cylinder has base radius r = 100 cm and height h = 100cm. Which of the following best describes how the volume of the cylinder will change if r increases to 101 cm and h decreases to 99 cm?

(A) Volume will decrease by approximately $3\pi(100)^2$ cubic cm.

(B) Volume will decrease by approximately $r\pi(100)^2$ cubic cm.

(C) Volume will increase by approximately $r\pi(100)^2$ cubic cm.

(D) Volume will increase by approximately $3\pi(100)^2$ cubic cm.

210. If P be the tangent plane to the surface $y^2z - 2xz^2 + 3x^2y = 2$ at the point $Q = (1, 1, 1)$ then which of the following points also lies in P?

(A) (–2, 4, 2) (B) (4, 5, –3)

(C) (6, –4, 3) (D) (3, –1, 5)

211. Suppose f, g, and h be functions of two variables that are differentiable everywhere such that $z = f(x, y)$, where $x = g(u, v)$ and $y = h(u, v)$. When $u = 0$ and $v = 1$, the values of x and y are 2 and 1, respectively. Suppose P_0 denote the point $(u, v) = (0, 1)$, and Q_0 denote the point $(x, y) = (2, 1)$. If $\frac{\partial f}{\partial x}|Q_0 = 11$, $\frac{\partial f}{\partial y}|Q_0 = -3$, $\frac{\partial g}{\partial u}|P_0 = 1$, $\frac{\partial h}{\partial u}|P_0 = -3$ and $\frac{\partial g}{\partial v}|P_0 = \frac{\partial h}{\partial v}|P_0 = 2$ then the value of $\frac{\partial z}{\partial v}|P_0$ is

(A) –21 (B) 16

(C) 12 (D) –10

212. The temperature at each point (x, y, z) in a room is given by the equation $T(x, y, z) = 9x^2 - 3y^2 + 6xyz$. A fly is currently hovering at the point (2, 2, 2). In the direction of which of the following vectors should the fly move in order to cool off as rapidly as possible?

(A) $-5\hat{i} - \hat{j} - 2\hat{k}$ (B) $5\hat{i} + \hat{j} + 2\hat{k}$

(C) $5\hat{i} + \hat{j} - 2\hat{k}$ (D) $5\hat{i} - \hat{j} - 2\hat{k}$

213. Suppose $f(x,y)$ be a function that is differentiable everywhere. At a certain point P in the xy-plane, the directional derivative of f in the direction of $\hat{i} - \hat{j}$ is $\sqrt{2}$ and the directional derivative of f in the direction of $\hat{i} + \hat{j}$ is $3\sqrt{2}$. What is the maximum directional derivative of f at P?

(A) $3\sqrt{2}$ (B) $\sqrt{2}$

(C) $2\sqrt{5}$ (D) $5\sqrt{2}$

214. Which of the following vectors is normal to the surface $\log(x + y^2 - z^3) = x - 1$ at the point where $y = 8$ and $z = 4$?

(A) $2\hat{i} - 3\hat{j} - \hat{k}$
(B) $\hat{i} - \hat{j} - 2\hat{k}$
(C) $\hat{i} + 2\hat{j}$
(D) $\hat{j} - 3\hat{k}$

215. The function $f(x, y) = x^3 + y^3 - 3xy$ has a local minimum at exactly one point, P. The value of f at P is

(A) -1
(B) 2
(C) -3
(D) 4

216. The minimum distance from the origin to the curve $3x^2 + 4xy + 3y^2 = 20$ is

(A) 1
(B) 2
(C) 3
(D) 4

217. The solution of the differential equation $\frac{dy}{dx} = x\sin(x)$, $y(0) = 1$ is

(A) $y = -x\cos(x) + \sin(x) + 1$
(B) $y = -x\cos(x) - \sin(x) + 1$
(C) $y = -x\cos(x) - \sin(x) - 1$
(D) $y = x\cos(x) + \sin(x) + 1$

218. The differential equation corresponding to the integral curves $y^4 + 4xy - x^4 = c$ is

(A) $(y^3 - x^3)dx + (y^3 + x)dy = 0$
(B) $(y^3 - x^3)dx + (y^3 + x^2)dy = 0$
(C) $(y - x^3)dx + (y^3 + x)dy = 0$
(D) $(y^3 - x^3)dx + (y^3 + x^3)dy = 0$

219. The solution of the differential equation $\frac{dy}{dx} = \frac{x(x-2)}{e^y}$ is

(A) $y = \log\left|x^3 - \frac{1}{2}x^2 + c\right|$
(B) $y = \log\left|\frac{1}{3}x^3 - x^2 + c\right|$
(C) $y = \log\left|-\frac{1}{3}x^3 + x^2 + c\right|$
(D) $y = \log\left|-\frac{1}{2}x^3 - x^2 + c\right|$

220. The integral curves corresponding to differential equation $(x^2 + y^2)dx - 2xydy$ is

(A) $x^3 - y^2 = cy$
(B) $x^2 - y^3 = cx$
(C) $x^2 - y^2 = cy$
(D) $x^2 - y^2 = cx$

221. The family of curves satisfying the equation $(1 - 2xy)dx + (4y^3 - x^2)dy = 0$ are

(A) $x^4 - x^2y + y = c$
(B) $x - x^2y + y^4 = c$
(C) $2x - xy^2 + y^4 = c$
(D) $2x - x^2y + y^4 = c$

222. The integral curve of the differential equation $\frac{dy}{dx}+\frac{x^2y}{x^3+y}=0$ that passes through the point (1, 1) is

(A) $4x^3y^3 + 3y^4 = 7$
(B) $4x^3y^3 - 3y^4 = 7$
(C) $4x^3y^3 + 3y^4 = -7$
(D) $4x^3y^3 - 3y^4 = -7$

223. The solution of the differential equation $\frac{dy}{dx}=5x-\frac{3y}{x}$ when $y(1)=2$ is

(A) $y = x^3 + x^{-3}$
(B) $y = x^3 + 2x^{-3}$
(C) $y = 2x^3 + x^{-3}$
(D) $y = x^3 + 3x^{-3}$

224. The general solution of the differential equation $y'' + 4y = 0$ is

(A) $y = c_1\cos 2x + c_2\sin 2x$
(B) $y = c_1\cos x + c_2\sin 2x$
(C) $y = c_1\cos 2x + c_2\sin x$
(D) $y = c_1\cos x + c_2\sin x$

225. The general solution of the differential equation $2y'' + 7y' = 4y$ is

(A) $y = c_1e^{x/2} + c_2e^{4x}$
(B) $y = c_1e^{x/2} + c_2e^{-4x}$
(C) $y = c_1e^{x} + c_2e^{-4x}$
(D) $y = c_1e^{-x} + c_2e^{4x}$

226. The general solution of the differential equation $y'' + 2y' + y = 0$ is

(A) $y = c_1e^{-x} + c_2x^2e^{-x}$
(B) $y = c_1xe^{-x} + c_2x^2e^{-x}$
(C) $y = c_1e^{-x} + c_2xe^{-x}$
(D) $y = c_1xe^{-x} + c_2x^3e^{-x}$

227. The general solution of the differential equation $y'''-y''-9y'+9y=0$ is

(A) $y = c_1e^{-2x} + c_2e^{2x} + c_3e^{x}$
(B) $y = c_1e^{-2x} + c_2e^{3x} + c_3e^{x}$
(C) $y = c_1e^{-2x} + c_2e^{2x} + c_3e^{-3x}$
(D) $y = c_1e^{-3x} + c_2e^{3x} + c_3e^{x}$

228. The general solution of the differential equation $y'' = x + y$ is

(A) $y = c_1e^{2x} + c_2e^{3x} - x$
(B) $y = c_1e^{x} + c_2e^{-x} - x$
(C) $y = c_1e^{2x} + c_2e^{-3x} - x$
(D) $y = c_1e^{-x} + c_2e^{-3x} - x$

229. If $y = f(x)$ is the solution of $\frac{dy}{dx}=\frac{x^2}{x^2+1}$ such that $y=0$ when $x=0$. The value of $f(1)$ is

(A) $1 - \log 2$
(B) $\frac{1}{4}(4 - \pi)$
(C) $1 + \log 2$
(D) None of the above

230. A population of bacteria grows at a rate proportional to the number present. After two hours, the population has tripled. After two more hours elapse, the population will have increased by a factor of k. What is the value of k?

(A) 6
(B) 8
(C) 9
(D) 27

231. Every curve in a certain family, $y = f(x, c)$, has the following property: the area of the region in the first quadrant bounded above by the curve from (0,0) to (x, y) and bounded below by the x–axis is one–third the area of the rectangle with opposite vertices at (0, 0) and (x, y). The function $f(x, c)$ is given by

(A) cx^3
(B) $cx^3 + x$
(C) $cx^3 - x$
(D) cx^2

232. The integral curve corresponding to the differential equation $\left(\frac{dy}{dx}\right)^2 = \frac{x}{y}\left(2\frac{dy}{dx} - \frac{x}{y}\right)$ is

(A) $y^3 - x^2 = cx$
(B) $y^2 - x^3 = cx^2$
(C) $y^3 - x^2 = cy$
(D) $y^2 - x^2 = c$

233. If a is a positive constant, let $y = f(x)$ be the solution of the equation $y''' - ay'' + a^2y' - a^3y = 0$ such that $f(0) = 1$, $f'(0) = 0$ and $f''(0) = a^2$. The number of positive values of x satisfying the equation $f(x) = 0$ are

(A) 0
(B) 1
(C) 2
(D) more than 2

234. Suppose $g : \mathbb{R} \to \mathbb{R}$ be a differentiable and integrable function. The integral curve of the differential equation $[y + g(x)]dx + [x - g(y)\backslash dy = 0$ that passes through the point (1,1) must also passes through

(A) (–1, –1)
(B) (0,0)
(C) (2,1/2)
(D) (1/2,2)

235. If $y = f(x)$ is the solution of $\frac{dy}{dx} + \frac{y}{x} = \sin x$ such that $f(\pi) = 1$ then the value of $f\left(\frac{\pi}{2}\right)$ is

(A) $2/\pi - 1$
(B) $2/\pi$
(C) $2/\pi + 1$
(D) $2/\pi + 2$

236. If $y = f(x)$ is the solution of $\frac{d^4y}{dx^4} = \frac{d^2y}{x^2}$ such that $f(0) = f'(0) = f''(0) = 0$ and $f'''(0) = -1$. The value of $f(x)$ is

(A) $x - \cosh x$
(B) $x + \cosh x$
(C) $x - \sinh x$
(D) $x + \sinh x$

237. The general solution of the equation $2y''' + 7y'' + 3y = 6$ is

(A) $y = 2x + c_1 + c_2e^{-x/2} + c_3e^{-3x}$

(B) $y = 2 + c_1 + c_2e^{-x/2} + c_3e^{-3x}$

(C) $y = x^2 + c_1 + c_2e^{-x/2} + c_3e^{-3x}$

(D) $y = 2x + c_1 + c_2e^{-x/2} + c_3e^{-2x}$

238. The equation $(3xy^2 - 5y)dx + (2x^2y - 3x)dy = 0$ has an integrating factor of the form $g(x, y) = x^my^n$ then its general solution is

(A) $x^4y^2 (0.5xy - 1) = c$

(B) $x^4y^2 (2xy - 1) = c$

(C) $x^5y^3 (2xy - 1) = c$

(D) $x^5y^3 (0.5xy - 1) = c$

239. For a curve in xy plane the slope is given by $\frac{1-2xy}{x^2+3y^2+1}$. The equation of the curve given that it passes through the point (1, 1).

(A) $\frac{1}{3}x^3 + 3xy^2 + x + y - xy^2 = \frac{13}{3}$

(B) $\frac{1}{3}x^3 + 3xy^2 - x + y - xy^2 = \frac{13}{3}$

(C) $xy^2 + y^3 + x - y = 2$

(D) $x^2y + y^3 - x + y = 2$

240. The general solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ is

(A) $e^{y/x} = cx$

(B) $e^{y/x} = cy$

(C) $e^{x/y} = cx$

(D) $e^{x/y} = cy$

241. Consider the family F of circles in the xy–plane, $(x-c)^2 + y^2 = c^2$, that are tangent to the y–axis at the origin. Which of the following gives the differential equation that is satisfied by the family of curves orthogonal to F?

(A) $y' = \frac{x}{x-y}$

(B) $y' = \frac{x}{y-x}$

(C) $y' = \frac{xy}{y-x}$

(D) $y' = \frac{2xy}{x^2-y^2}$

242. Suppose $g(x, y)$ be the function defined for all x and all nonzero y such that the differential equation $(\sin xy)dx + g(x, y)dy = 0$ is exact and $g(0, y) = 0$ for all $y \neq 0$. What is $g(x, 1)$?

(A) $\sin x + \cos x - 1$

(B) $x\sin x + \cos x - 1$

(C) $x\sin x + \cos x + 1$

(D) $x\sin x + \cos x$

243. If $w = f(x, y)$ is a solution of the partial differential equation $2\frac{\partial w}{\partial x} - 3\frac{\partial w}{\partial y} = 0$ then w could equal

(A) $(2x - 3y)^6$

(B) $\sin[\log(3x - 2y)]$

(C) $e^{\tan^{-1}(3x+2y)}$

(D) $\sqrt{2x + 3y}$

244. There is only one integer, x, between 100 and 200 such that integer pair (x, y) satisfies the equation $42x + 55y = 1$. What's the value of x in this integer pair?

(A) 127

(B) 148

(C) 158

(D) 167

245. Let L be the least common multiple of 1001 and 10101. What's the sum of the digits of L?

(A) 6

(B) 11

(C) 17

(D) 22

246. Let x_1 and x_2 be the two smallest positive integers for which the following statement is true: "$85x - 12$ is a multiple of 19." Then $x_1 + x_2 =$

(A) 27

(B) 31

(C) 38

(D) 47

247. If x, y and z are positive integers such that $4x - 5y + 2z$ is divisible by 13, then which one of the following must also be divisible by 13?

(A) $6x - 10y - z$

(B) $x - y - 2z$

(C) $-7x + 12y + 3z$

(D) $-5x + 3y - 4z$

248. When expressed in its usual decimal notation, the number 100! (that is, 100 factorial) ends in how many consecutive zeros?

(A) 20

(B) 24

(C) 30

(D) 32

249. How many generators does the group $(\mathbb{Z}_{24}, +)$ have?

(A) 2

(B) 6

(C) 8

(D) 10

250. Which one of the following groups is cyclic?

(A) $\mathbb{Z}_2 \times \mathbb{Z}_4$

(B) $\mathbb{Z}_2 \times \mathbb{Z}_6$

(C) $\mathbb{Z}_3 \times \mathbb{Z}_4$

(D) $\mathbb{Z}_3 \times \mathbb{Z}_6$

251. If G is a group of order 12, then G must have a subgroup of all of the following orders except

(A) 2
(B) 3
(C) 4
(D) 6

252. How many subgroups does the group $\mathbb{Z}_3 \oplus \mathbb{Z}_{16}$ have?

(A) 6
(B) 10
(C) 12
(D) 20

253. If $S = \{a \in \mathbb{R}^+ : a \neq 1\}$, with the binary operation • defined by the equation $a \bullet b = a^{\log b}$ (where $\log b = \log_e b$), then $(S, \bullet)$ is a group. What is the inverse of $a \in S$?

(A) $e/\log a$
(B) $e^{-\log a}$
(C) $e^{\log(1/a)}$
(D) $e^{(1/\log a)}$

254. Which of the following are subgroups of $GL(2, \mathbb{R})$, the group of invertible 2×2 matrices (with real elements) under matrix multiplication?

I. $T = \{A \in GL(2, \mathbb{R}) : \det A = 2\}$
II. $U = \{A \in GL(2, \mathbb{R}) : A \text{ is upper triangular}\}$
III. $V = \{A \in GL(2, \mathbb{R}) : \text{trace}(A) = 0\}$

(A) I and II only
(B) II only
(C) II and III only
(D) III only

255. Let p and q be distinct primes. How many (mutually nonisomorphic) Abelian groups are there of order p^2q^4?

(A) 6
(B) 8
(C) 10
(D) 12

256. Let G be the group generated by the elements x and y and subject to the following relations: $x^2 = y^3$, $y^6 = 1$, and $x^{-1}yx = y^{-1}$. Express in simplest form the inverse of the element $z = x^{-2}yx^3y^3$ is

(A) xy^2
(B) xy
(C) yx
(D) y^2x

257. Let H be the set of all group homomorphisms $\phi : \mathbb{Z}_3 \rightarrow \mathbb{Z}_6$. How many functions does H contain?

(A) 1
(B) 2
(C) 3
(D) 4

258. Let G be a group of order 9, and let e denote the identity of G. Which one of the following statements about G cannot be true?

(A) There exists an element x in G such that $x \neq e$ and $x^{-1} = x$.

(B) There exists an element x in G such that $x \neq e$ and $x^2 = x^5$.

(C) There exists an element x in G such that $\langle x \rangle$ has order 3.

(D) G is cyclic.

259. Let R be a ring; an element x in R is said to be idempotent if $x^2 = x$. How many idempotent elements does the ring $\mathbb{Z}_{20}$ contain?

(A) 2 (B) 4

(C) 5 (D) 8

260. Which of the following rings are integral domains?

I. $\mathbb{Z} \oplus \mathbb{Z}$

II. $\mathbb{Z}_p$, where p is a prime

III. $\mathbb{Z}_{p^2}$, where p is a prime

(A) I and II only (B) II only

(C) II and III only (D) III only

261. Which one of the following rings does not have the same number of units as the other three?

(A) $\mathbb{Z} \oplus \mathbb{Z}$ (B) $\mathbb{Z} \oplus \mathbb{Z}_3$

(C) $\mathbb{Z} \oplus \mathbb{Z}_5$ (D) $\mathbb{Z} \oplus \mathbb{Z}_6$

262. How many elements x in the field $\mathbb{Z}_{11}$ satisfy the equation $x^{12} - x^{10} = 2$?

(A) 1 (B) 2

(C) 3 (D) 4

263. Which of the following are subfields of $\mathbb{C}$

I. $K_1 = \{a + b\sqrt{\frac{2}{3}} : a, b \in \mathbb{Q}\}$

II. $K_2 = \{a + b\sqrt{2} : a, b \in \mathbb{Q} \text{ and } ab < \sqrt{2}\}$

III. $K_3 = \{a + bi : a, b \in \mathbb{Z} \text{ and } i = \sqrt{-1}\}$

(A) K_1 only (B) K_1 and K_2 only

(C) K_3 only (D) K_1 and K_3 only

264. Let's say that the personal code for your ATM card consists of six characters, each of which can be any letter or numerical digit. How many possible personal codes are there?

(A) 36!
(B) 36!/(6! × 30!)
(C) 36^6
(D) 36!/30!

265. In the symmetric group S_5, the permutation (1, 3, 5, 4) is equal to

(A) (3, 5, 4, 1)
(B) (5, 4, 1, 3)
(C) (4, 1, 3, 5)
(D) All of the above.

266. In a horse race consisting of 8 horses, how many different win-place-show (the top three) finishing orders are there?

(A) 336
(B) 236
(C) 562
(D) 486

267. How many 3-element subsets does a set containing 9 elements have?

(A) 82
(B) 83
(C) 84
(D) 85

268. In how many ways can we write the number 4 as the sum of 5 nonnegative integers?

(A) 60
(B) 70
(C) 80
(D) 90

269. Each of the first ten positive integers is written on a slip of paper, and the ten slips are tossed into a hat. What's the probability that someone will pull out a prime number?

(A) 2/5
(B) 5/8
(C) 3/4
(D) 1/2

270. Two points x and y are selected at random in the interval [0,1]. What's the probability that the product xy will be less than 2 ?

(A) $\frac{1}{2}(1 - \log 2)$
(B) $\frac{1}{3}(1 + \log 2)$
(C) $\frac{1}{2}(1 + \log 2)$
(D) $\frac{1}{3}(1 - \log 2)$

271. If A and B are events in a probability space then which of the following statements are true?

(A) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(B) A and B are independent if and only if $P(A \cap B) = P(A)\,P(B)$.

(C) If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$.

(D) All of the above.

272. A gambler throws two fair dice twice. Let A be the event that the first toss is a 7 or an 11, and let B be the event that the second toss is an 11. What's $P(A \text{ or } B)$?

(A) 42 / 161
(B) 43 / 162
(C) 44 / 163
(D) 45 / 164

273. Let E be an algebra of sets on S that contains the sets A and B, on which a probability measure P is defined. Given that the events A and B^c are independent, where B^c represents the event that B does not occur, $P(A) = 1/4$, and $P(B) = 1/3$, what is $P(A \text{ or } B^c)$?

(A) 1/4
(B) 1/2
(C) 3/4
(D) None of the above

274. Let X be a random variable whose distribution function is

$f_x(t) = \begin{cases} 0 \text{ for } t < 0 \\ 1 - e^{-2t} \text{ for } t \geq 0 \end{cases}$ The value of $P(X \leq 1/2)$ and $P(1/3 < X \leq 2/3)$ are

(A) $(1 - e)$ and $(e^{2/3} - e^{4/3})$
(B) $(1 - e^{-1})$ and $(e^{-2/3} - e^{-4/3})$
(C) $(1 - e)$ and $(e^{2/3} - e^{-4/3})$
(D) $(1 - e^{-1})$ and $(e^{-2/3} - e^{4/3})$

275. A company hires a marketing consultant who determines that the length of time (in minutes) that a consumer spends on the company's Web site is a random variable X whose probability density function is

$f_x(t) = \begin{cases} 0 \text{ for } t < 0 \\ \frac{1}{6} e^{-t/6} \text{ for } t \geq 0 \end{cases}$ What is the probability that a consumer will spend more than 10 minutes on the company's Web site?

(A) $e^{-5/3}$
(B) e^{-3}
(C) $e^{-3/5}$
(D) e^{-5}

276. Let X be a random variable whose probability density function is:

$$f(x)=\begin{cases}0 \text{ for } x\le 0\\ \frac{1}{4}x^3 \text{ for } 0<x<2\\ 0 \text{ for } x\ge 2\end{cases}$$ The standard deviation of X is

(A) $2\sqrt{3}/5$ (B) $2\sqrt{5}/7$

(C) $2\sqrt{6}/15$ (D) $2\sqrt{7}/9$

277. If A be the subset (1, 2] in $\mathbb{R}$ then the exterior of A is

(A) (1, 2) (B) [1, 2]

(C) $(-\infty, 1) \cup (2, \infty)$ (D) None of the above

278. If A be the subset $(0, 1) \cup (1, 2)$ in $\mathbb{R}$ then the boundary of A is

(A) $(0, 1) \cup (1, 2)$ (B) $(-\infty, 0) \cup (2, \infty)$

(C) [0, 2] (D) {0, 1, 2}

279. If A be the subset $(0, 1) \cup \{2\} \cup [3, 4]$ in $\mathbb{R}$ then the closure of A is

(A) $(0, 1) \cup (3,4)$ (B) {0,1, 2, 3,4}

(C) $[0, 1] \cup \{2\} \cup [3, 4]$ (D) $[0, 1] \cup [3, 4]$

280. If X be a nonempty set, and let $\mathbb{B}$ be a collection of subsets of X then which of the following is true?

I. For every x in X, there is at least one set $B \in \mathbb{B}$ such that $x \in B$.

II. If B_1 and B_2 are sets in B and $x \in B_1 \cap B_2$, then there exists a set $B_3 \in \mathbb{B}$ such that $x \in B_3 \subseteq B_1 \cap B_2$.

(A) I only (B) II only

(C) both I and II (D) None of the above

281. Which of the following subsets of $\mathbb{R}^2$ are open in the product topology?

(A) The interior of the unit circle (boundary not included)

(B) The line $y = x$

(C) The set $(1, 2] \times (1, 2)$

(D) None of the above

282. If $f(z) = z - \tan^{-1}z$ then the solution of the equation $f'(z) = \frac{4}{3}$ is

(A) $z = \pm 1 - i$ (B) $z = 1 \pm i$

(C) $z = 1 \pm 2i$ (D) $z = \pm 2i$

283. If C is the circle $|z - i| = 1/2$, oriented counterclockwise, the value of the integral is $\oint_C \frac{z+1}{z(z-i)^2}\, dz$ is

(A) $\pi i/2$ (B) $\pi i/4$

(C) $2\pi i$ (D) $3\pi i/4$

284. Let $P(x) = x^3 + \frac{4}{3}x^2 - \frac{59}{9}x + 2$. Perform two iterations of the bisection method to approximate the root closest to $x = 0$. The root after two iteration is

(A) 3/8 (B) 1/4

(C) 2/3 (D) 4/9

285. For which of the following intervals does $P(x) = 4x3 - 4x2 - 33x + 45$ have a zero?

(A) [–2, –1] (B) [–1, 0]

(C) [0, 1] (D) [2, 3]

286. A fair die is tossed twice. About how many times would you expect to roll 3 or greater?

(A) 2 (B) 4/3

(C) 1 (D) 1/2

287. Which of the following sets is not countably infinite?

(A) $\mathbb{Q}$, the set of rational numbers.

(B) $\mathbb{Z}$, the set of all integers

(C) $\mathbb{Q}^c$, the set of all irrational numbers

(D) $\mathbb{Z}^+$, the set of positive integers

288. What is the coefficient of the $(z - 2)^{-1}$ term in the Laurent series for $f(z) = 1/(z - 5)$ centered at $z = 2$?

(A) 81
(B) 27
(C) 9
(D) 1

289. Let $g(x)$ be a polynomial function whose derivative is continuous and non zero on the interval $[a, b]$. Suppose there exists a y on this same interval such that $g(y)=0$. Let x_0 be an arbitrary x–value in the interval. Then x_1 is the x–intercept of the line tangent to $g(x)$ at x_0. For each subsequent n, x_n is the x–intercept of the line tangent to $g(x)$ at x_{n-1}. Which formula best approximates the root of $g(x)$ using the method described above?

(A) $x_{n+1} = x_n - g(x)/g^n(x)$
(B) $x_{n+1} = x_n - g'(x)/g''(x)$
(C) $x_{n+1} = x_n + g(x)/g'(x)$
(D) $x_{n+1} = x_n - g(x)/g'(x)$

290. If $x^2 = 40$, use Newton's method twice to approximate the value of x to three decimal places to get the approximate x as

(A) 6.325
(B) 6.326
(C) 6.327
(D) 6.328

291. The Laurent series expansions of the function $f(z) = \dfrac{1}{z-3}$ that is valid in the annulus $|z - 4| > 1$ is

(A) $\sum_{n=1}^{\infty}(4-z)^{-n-1}$
(B) $\sum_{n=0}^{\infty}(-1)^n(z-4)^{-n}$
(C) $\sum_{n=1}^{\infty}(-1)^n(z-4)^{-n-1}$
(D) $\sum_{n=0}^{\infty}(-1)^n(z-4)^{-n-1}$

292. A teacher is assigning 6 students to one of three tasks. She will assign students in teams of at least one student, and all students will be assigned to teams. If each task will have exactly one team assigned to it, then which of the following are possible combinations of teams to tasks?

I. 90 II. 60 III. 45

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only

293. Which of the following is the solution set of the inequality $x + \frac{6}{x} > 5$?

(A) (0, 2) ∪ (3, ∞)
(B) (0, 1) ∪ (2, ∞)
(C) (–∞, 2) ∪ (3, ∞)
(D) (0, 2) ∩ (3, ∞)

294. Which of the following sets in $\mathbb{R}^2$ is closed?

(A) (2, 5] × [1, 3)
(B) [2, 5] × (1, 3)
(C) (2, 5) × [1, 3]
(D) [2, 5] × [1, 3]

295. What are the complex roots of the equation $e^{2z} = i$?

(A) $2i\left(\frac{\pi}{2} + 2n\pi\right)$
(B) $\frac{i}{2}\left(\frac{\pi}{2} + 2n\pi\right)$
(C) $\frac{i}{2}\left(\frac{\pi}{2} + n\pi\right)$
(D) $\frac{i}{2}\left(\frac{\pi}{4} + n\pi\right)$

296. In the complex plane, the set of all points that satisfy the equation $\bar{z}^2 = z^2$ is

(A) a circle.
(B) a point.
(C) a line.
(D) two lines.

297. Which of the following is a harmonic conjugate $u(x, y)$ of the harmonic function $v = x–3x^{2y} + y^3$?

(A) $–x^3 + 3xy^2 – y$
(B) $x^3 + 3xy^2 – y$
(C) $x^3 – 3xy^2 – y$
(D) $x^3 – 3xy^2 + y$

298. What is the polar form of a complex number equal to $(i – \sqrt{3})^6$?

(A) $–2^6(1 + i)$
(B) $–2^6(1 – i)$
(C) $–2^6$
(D) 2^6

299. If $M = (– 1, 3] \cup [7, 8)$, what is the Lebesgue measure of M?

(A) 0
(B) 5
(C) 9
(D) –1

300. Let $f(z) = (5x – 3y) + iv(x, y)$ be an analytic function where $v(x, y)$ is real valued function and $x, y \in \mathbb{R}$. If $v(4, 1) = 7$ then $v(3, 2)$ is

(A) – 9
(B) 9
(C) –1
(D) 1